

Ableitung mehrfach verketteter Funktionen

1)

$$f(x) = \left[\sin\left(\frac{1}{2}x - 4\right) \right]^3$$

$$\begin{aligned} f'(x) &= 3 \left[\sin\left(\frac{1}{2}x - 4\right) \right]^2 \cdot \cos\left(\frac{1}{2}x - 4\right) \cdot \frac{1}{2} \\ &= \frac{3}{2} \left[\sin\left(\frac{1}{2}x - 4\right) \right]^2 \cdot \cos\left(\frac{1}{2}x - 4\right) \end{aligned}$$

2)

$$\begin{aligned} f(x) &= \frac{x^2}{\cos(\sqrt{x})} = \frac{x^2}{1} \cdot \frac{1}{\cos(\sqrt{x})} \\ &= x^2 \cdot [\cos(\sqrt{x})]^{-1} \end{aligned}$$

$$\begin{aligned} f'(x) &= 2x \cdot [\cos(\sqrt{x})]^{-1} + x^2 \cdot (-1) \cdot [\cos(\sqrt{x})]^{-2} \cdot (-\sin(\sqrt{x})) \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{2x}{\cos(\sqrt{x})} + \frac{x^2 \cdot (\sin(\sqrt{x}))}{[\cos(\sqrt{x})]^2 \cdot 2\sqrt{x}} \\ &= \frac{2x \cdot 2\sqrt{x} \cdot \cos(\sqrt{x})}{\cos(\sqrt{x}) \cdot 2\sqrt{x} \cdot \cos(\sqrt{x})} + \frac{x^2 \cdot (\sin(\sqrt{x}))}{[\cos(\sqrt{x})]^2 \cdot 2\sqrt{x}} \\ &= \frac{4x\sqrt{x} \cdot \cos(\sqrt{x})}{[\cos(\sqrt{x})]^2 \cdot 2\sqrt{x}} + \frac{x^2 \cdot (\sin(\sqrt{x}))}{[\cos(\sqrt{x})]^2 \cdot 2\sqrt{x}} \\ &= \frac{4x\sqrt{x} \cdot \cos(\sqrt{x}) + x^2 \cdot (\sin(\sqrt{x}))}{2\sqrt{x} \cdot \cos^2(\sqrt{x})} \end{aligned}$$